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ABSTRACT The work reported here is taken verbatim from a proposal for the study and design of a computer aided sonar echo classification system. In the proposal, the discussion of a signal processing and waveform design was felt to be of sufficient general nature to be useful to others working in related fields. Sid Applebaum, who has since terminated his			
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#### FORWARD

The work reported here is taken verbatim from a proposal for the study and design of a computer aided sonar echo classification system. In the proposal, the discussion of signal processing and waveform design was felt to be of sufficient general nature to be useful to others working in related fields. Sid Applebaum, who has since terminated his services with the Company, is the sole author of this work.

## SIGNAL PROCESSING AND WAVEFORM DESIGN

The intent here is to discuss some theoretical aspects of the signal processing and waveform design areas. In addition, some recent findings on "noise" waveforms shall be presented. Noise or random waveforms are especially attractive for target classification because they combine the attributes of long duration and wide bandwidth, thus offering the possibility of simultaneous resolution in both range and doppler. Noise waveforms are attractive, too, because techniques have been developed for efficiently generating and processing such waveforms.

It is convenient for this discussion to group reflectors of sound energy into two classes - point targets and extended targets. A point target is one whose physical dimensions are small compared to a wavelength of the transmitted carrier frequency, while an extended target has large dimensions compared to a wavelength. If acoustic radiation is incident on an extended target having a complicated shape, it is found that the main contribution to the scattered field is due to "specular" points, or patches, occupying a relatively small part of the illuminated surface of the object. An analogous phenomenon of common experience occurs in the "spotty" reflection of the sun or moon from a wavy water surface.

The number and location of the specular points observed depends upon the target's shape and size, and on its orientation with respect to the observer. Objects of simple geometric shape will exhibit a small number of specular points. A sphere, for example, has only one specular point and, therefore, will look like a point target even though it may be large compared to a wavelength.

Extended targets may, therefore, be considered to be a group or cluster of point targets. The distribution of these point targets in range and amplitude will be characteristic of the target shape, and, hence, may be used to recognize or classify the target. The function of signal processing in classification is to obtain this distribution accurately and in as much detail as the transmitted waveform permits. The waveform should be capable of providing high resolution information, preferably enough to individually resolve at least a few of the target's major specular points. If a sonar system has poor range resolution, all of the target's specular points may appear as a single point target, and there will be little information on which to base a classification decision.

Target motion can be of considerable help in classification. If a rigid extended target is moving, the signals from all of its specular points will display the same doppler shift. Doppler shift can thus serve as a tag to identify the specular point returns from a single extended target. With sufficient doppler resolution, the group of specular points belonging to one moving extended target can be separated from the group belonging to another stationary target, even though both targets may be at the same range. Doppler information can also help recognize "nonrigid" targets like a school of fish, on the basis of the amount of doppler spread of the returned echoes.

Since all targets can be considered as groups of point targets, the waveform design and signal processing portions of the classification problem reduces, in effect, to the problem of detecting, resolving, and estimating the parameters of point targets. A great deal of work has been done in this area. A brief review of the major theoretical results, and their application to the problem of sonar echo classification is presented in the remainder of this section.

#### A. OPTIMUM SIGNAL PROCESSING

A discussion of signal processing and waveforms is necessarily mathematical. To save time and space complex signal notation shall be used.\* In complex notation, the transmitted signal may be represented as

$$(2 E_T)^{1/2} u(t) \exp j 2\pi f_c t \quad (1)$$

where  $u(t)$ , a complex waveform is normalized so that

$$\int |u(t)|^2 dt = 1 \quad (2)$$

$E_T$  = transmitted energy, and

$f_c$  = the carrier frequency.

(Unless otherwise specified, the limits on all integrals are to be taken from  $-\infty$  to  $+\infty$ .)

If there is a stationary point target present at a round trip range delay of  $r$  seconds, the signal received from the target may be represented by

$$s(t) \exp j 2\pi f_c (t - r) \quad (3)$$

---

\*See P.M. Woodward, "Probability and Information Theory with Applications to Radar", Pergamon, London 1953.

where

$$s(t) = (2 E_S)^{1/2} u(t - r) \quad (4)$$

$E_S$  is the received signal energy.  $E_S$  depends upon the transmitted energy, on the cross section of the target, and on attenuation factors that are functions of range. It has been assumed that the signal bandwidth is sufficiently narrow so that dispersion in the medium produces negligible distortion of the signal waveform,  $u(t)$ .

If the point target is moving with a constant, positive range rate, the received signal will exhibit a corresponding doppler frequency shift, say  $-f_d$ . The complex envelope of the received signal will then be of the form

$$s(t) = (2 E_S)^{1/2} u(t - r) \exp -j 2\pi f_d (t - r) \quad (5)$$

It is apparent that the signal from any point target is completely specified by three parameters - energy, range delay, and doppler frequency shift. All of the classification information available from a resolved point target echo is contained in the parameters  $E_S$ ,  $r$ , and  $f_d$ , or functions of these three parameters. The information available from a cluster of unresolved signal echoes is completely defined by the distribution of received energy in range delay and doppler frequency. Both types of information can be provided by a signal processor that estimates the received energy  $E_S$  continuously as a function of both  $r$  and  $f_d$ .

The type of signal processing and waveform to use for optimum parameter estimation depends on the environment in which the system operates. In addition to the signal from the target of interest, the receiver will pick up signals from other sources, such as neighboring targets, reverberation, multipath signals, and, of course, random thermal-type noise. These environmental factors determine the threshold detectability of a target and the accuracy with which its parameters can be estimated.

The situation in which random noise is the major environmental factor shall be considered first. The signal processing part of the classification problem then reduces to the well-known problem of estimating the parameters of a signal in additive Gaussian noise. The waveform received in this case contains two components, and, omitting the carrier frequency factors, may be represented as

$$v(t) = s(t) + n(t) \quad (6)$$

where  $n(t)$  = complex noise modulation.

It has been shown in the literature that the optimum signal processor for this situation is one that cross-correlates the received waveform,  $v(t)$ , with a stored replica of  $u(t)$ , the transmitted waveform. The cross-correlation must be performed for all range and doppler shifts of interest. The optimum signal processor can be defined more precisely as one that generates a two-dimensional array of data,  $X(\hat{r}, \hat{f}_d)$  where

$$X(\hat{r}, \hat{f}_d) = \int v(t) u^*(t - \hat{r}) e^{j 2\pi \hat{f}_d t} dt \quad (7)$$

The cross-correlation can be performed "actively" by storing  $u(t)$  at the receiver, multiplying by  $u(t - \hat{r})$  (with  $r$  varying over the range of interest), and then integrating the product in some form of spectrum analyzer. The Deltic-VICI technique is one method for accomplishing this. The cross-correlation can also be performed "passively" in a bank of "matched filters". In this case the receiver consists of an array of filters with each one conjugate-matched to a frequency-shifted equivalent of the transmitted waveform,  $u(t)$ . The transfer function of each filter will be of the form

$$U^*(f - f_c - \hat{f}_d)$$

where  $U(f)$  is the Fourier transform of  $u(t)$ , that is

$$U(f) = \int u(t) e^{-j 2\pi f t} dt \quad (8)$$

The number of filters required in a "matched-filter" receiver is approximately equal to the total doppler surveillance band divided by the system doppler resolution, which depends on the duration of the waveform,  $u(t)$ . In the matched-filter receiver the doppler shift of the signal is determined by noting which filter has the largest output, range is determined by the time at which the peak output occurs, and energy is determined by measuring the amplitude of the peak output.

The cross-correlating or matched-filter receiver is characterized by the property that it maximizes the ratio of peak signal output to rms noise output. The mean square noise output will be the same for all values of  $r$ ,  $f_d$  and is given by

$$\begin{aligned} \overline{X_n^2} &= N_o \int |U(f)|^2 df \\ &= N_o \int |u(t)|^2 dt \end{aligned} \quad (9)$$

where  $N_o$  is the (one-sided) power density of the noise (watts per cycle).

The "signal" portion of the signal processor output will be

$$X_S(\hat{r}, \hat{f}_d) = \int s(t) u^*(t - \hat{r}) e^{j 2\pi \hat{f}_d t} dt \quad (10)$$

If  $s(t)$  is the signal returned by a point target at a range delay  $r$ , moving so as to produce a doppler shift of  $-f_d$ , Equation (10) becomes

$$X_S(\hat{r}, \hat{f}_d) = (2 E_S)^{1/2} \int u(t - r) u^*(t - \hat{r}) e^{-j 2\pi (\hat{f}_d - \hat{f}_d) t} dt \quad (11)$$

or

$$X_S(\hat{r}, \hat{f}_d) = (2 E_S)^{1/2} A_u(r - \hat{r}, \hat{f}_d - \hat{f}_d) \quad (12)$$

where

$$A_u(\tau, \nu) = \int u(t) u^*(t + \tau) e^{-j 2\pi \nu t} dt \quad (13)$$

The peak signal output will occur when  $\hat{r} = r$ , and  $\hat{f}_d = \hat{f}_d$ , at which point,

$$\begin{aligned} X_S(r, \hat{f}_d) &= (2 E_S)^{1/2} A_u(0,0) \\ &= (2 E_S)^{1/2} \end{aligned} \quad (14)$$

since  $A_u(0,0) = 1$ , from Equations (2) and (13).

The ratio of peak signal output to rms noise output from Equations (9) and (14) is therefore

$$\frac{\text{Peak Signal}}{\text{rms Noise}} = \frac{(2 E_S)^{1/2}}{N_o^{1/2}} \quad (15)$$

This is an important fundamental result showing that the output signal-to-noise ratio depends only on the energy in the received waveform and not at all on its shape. Since the peak power that can be transmitted is limited by cavitation, high signal-to-noise ratios can only be obtained by using waveforms of long duration. Long waveforms, however, have poor range resolution unless some form of coding or modulation is used. For target classification, it is desirable to have both high signal-to-noise ratio and good range resolution. Long coded waveforms must therefore be considered, that is, waveforms that have a large bandwidth-time product.

The resolution capabilities of a waveform in a matched filter system are determined by the function  $A_u(\tau, \nu)$  defined in Equation (13).

The function  $A_u$  is called the ambiguity function of the waveform  $u(t)$ ; it gives the distribution of "interference" or "cross-talk" due to a single point target. The ambiguity function is a sort of aperture or window with which targets distributed in range and velocity are viewed. From the definition, Equation (13), the following important relations may be derived

$$A_u(\tau, \nu) = \int U(f + \nu) U^*(f) e^{-j 2\pi f \tau} df \quad (16)$$

$$|A_u(\tau, \nu)|^2 \leq |A_u(0, 0)|^2 \quad (17)$$

$$|A_u(-\tau, -\nu)|^2 = |A_u(\tau, \nu)|^2 \quad (18)$$

and

$$\iint |A_u(\tau, \nu)|^2 d\tau d\nu = |A_u(0, 0)|^2 = 1 \quad (19)$$

Equation (16) expresses the ambiguity function in terms of  $U(f)$ , the Fourier transform of  $u(t)$ . Equation (17) shows that the ambiguity function attains its peak value at the "target position"  $(\tau, \nu = 0, 0)$ , verifying, in part, the optimality of matched filtering. Equation (18) reveals an important reciprocal property of the ambiguity function; it can be shown from Equation (18) that the cross-talk power at the point  $\tau, \nu$ , due to a target at  $0, 0$ , equals the cross-talk power at  $0, 0$ , due to an equal size target at the point  $\tau, \nu$ . Equation (19) is known as the Uncertainty Principle; it says that the total integrated cross-talk power generated by a point target is a constant independent of the shape of the waveform  $u(t)$ .

Although the total cross-talk power or ambiguity is invariant with the choice of waveform,  $u(t)$ , the manner in which it is distributed in range and doppler depends very much on the waveform coding. Ideally it would be preferred for  $A_u(\tau, \nu)$  to consist of a single sharp central spike with the unavoidable excess ambiguity uniformly distributed as low-level "side lobes". Such an ambiguity function has been referred to as a "thumb-tack" function. The dimensions of the central "spike" of an ambiguity function are approximately  $1/B$  in range ( $\tau$ ), and  $1/T$  in doppler ( $\nu$ ), where  $B$  and  $T$  are the bandwidth and time duration of the waveform, respectively.

The response of a matched filter receiver to an input signal pulse is shown in Figure 1. The array of outputs depicted defines the ambiguity function of the pulse waveform. The nature of the ambiguity functions of a number of different waveforms is indicated in Figure 2, in the form of "ambiguity diagrams". In these diagrams, only the regions of high ambiguity are shown. The side lobes of the ambiguity functions will spread over  $2T$  in range ( $\tau$ ) and  $2B$  in doppler ( $\nu$ ). The ambiguity diagrams of a short and long unmodulated pulse are given in Figures 2a and 2b, respectively. The short pulse spreads its ambiguity in doppler while the long pulse ambiguity is spread in range. As a result, the short pulse has good range resolution but poor doppler resolution, whereas the converse is true of the long pulse. The ambiguity diagrams of a linear frequency modulated pulse are shown in Figure 2c. The region of high ambiguity in this case has its major axis along a diagonal extending deep into the first and third quadrants. With the linear FM waveform, resolution is poor for targets whose range-velocity separations lie along the diagonal.

#### B. PERFORMANCE IN REVERBERATION

Up to this point, random noise has been considered as the only environmental factor limiting performance. Usually, however, reverberation is at least as important as random noise in determining sonar performance. Reverberation may be regarded as an extended target consisting of a large number of small randomly distributed point targets or scatterers. Each point scatterer will produce a signal of the form of Equation (12) at the output of a matched filter receiver. The point scatterers are assumed to be too densely distributed to be individually resolved.

Because of the random nature of reverberation, the average reverberation power at the output of a matched filter receiver is obtained by summing the individual signals, power-wise. If  $w(r, f_d) dr df_d$  is the energy received from the point scatterers located in the range interval  $r, r + dr$  and producing doppler frequency shifts between  $f_d$  and  $f_d + df_d$ , the mean power out of the matched filter receiver due to reverberation will be

$$\overline{x_R^2}(\hat{r}, \hat{f}_d) = 2 \iint w(r, f_d) \cdot |A_u(r - \hat{r}, f_d - \hat{f}_d)|^2 dr df_d \quad (20)$$

In order to proceed further with this expression, the detailed character of the function  $w(r, f_d)$  must be known. For most cases of interest a reasonable assumption is that  $w$  varies sufficiently slowly with range so that it may be regarded as a

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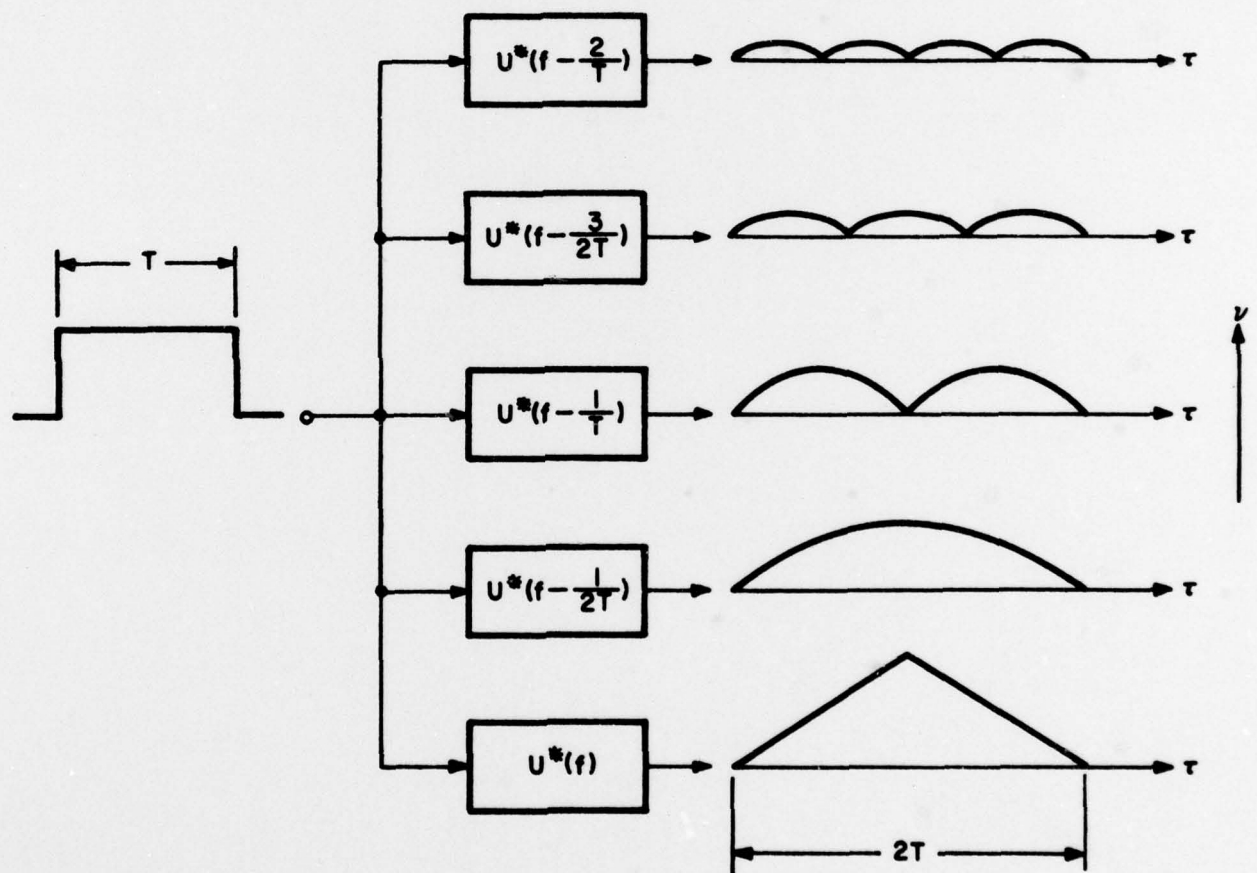
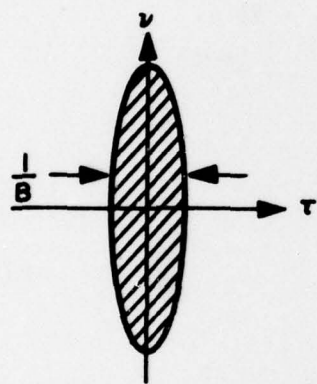
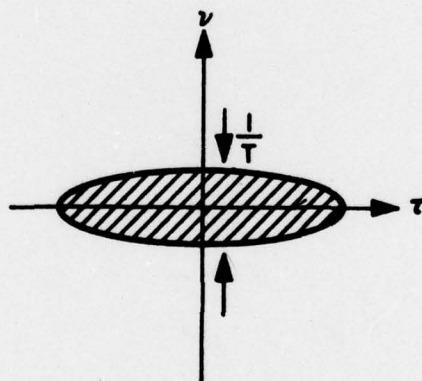


Figure 1. Response of Matched Filter Receiver

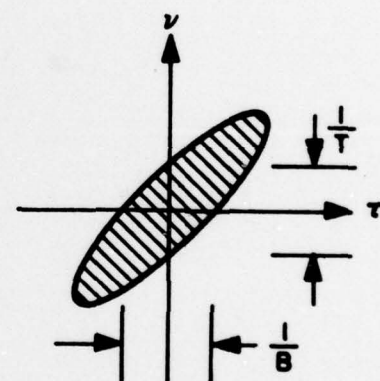
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(A) SHORT PULSE



(B) LONG PULSE



(C) LINEAR FM

Figure 2. Ambiguity Diagrams

function of  $f_d$  only in the integral shown in Equation (20). Formally, this assumes that, in the vicinity of  $r = \hat{r}$ ,

$$w(r, f_d) = W(r) \Phi(f_d, \hat{r}) \quad (21)$$

where  $W$  and  $\Phi$  are slowly varying functions of  $\hat{r}$ , and  $\Phi$  is assumed to be normalized so that

$$\int \Phi(f_d, r) df_d = 1 \quad (22)$$

The function  $\Phi$  describes the distribution of the reverberation energy in the doppler dimension. Usually it will be peaked near zero doppler (assuming that any ship's motion is compensated). Substituting Equation (21) in Equation (20)

$$\overline{x_R^2}(\hat{r}, \hat{f}_d) = 2 W(\hat{r}) \iint \Phi(f_d, \hat{r}) |A_u(r - \hat{r}, f_d - \hat{f}_d)|^2 dr df_d \quad (23)$$

$$= 2 W(\hat{r}) \int \Phi(f_d, \hat{r}) G_u(f_d - \hat{f}_d) df_d \quad (24)$$

where

$$G_u(\nu) = G_u(-\nu) = \int |A(\tau, \nu)|^2 d\tau \quad (25)$$

When the distribution of reverberation energy is highly concentrated around zero doppler, the function  $\Phi$  becomes a "delta" function

$$\Phi = \delta(f_d) \quad (26)$$

In this case Equation (24) reduces to

$$\overline{x_R^2}(\hat{r}, \hat{f}_d) = 2 W(\hat{r}) G_u(\hat{f}_d) \quad (27)$$

These equations, (24) and (27), describe the distribution of reverberation power at the output of a matched filter receiver. If there is a point target of interest at  $r = \hat{r}$ , having a doppler shift  $f_d = \hat{f}_d$ , and returning a signal with energy  $E_s$ , the ratio of reverberation power to peak signal power at the receiver output will be

$$\frac{\overline{x_R^2}}{x_S^2} = \frac{W(\hat{r})}{E_s} \int \Phi(f_d, \hat{r}) G_u(f_d - \hat{f}_d) df_d \quad (28)$$

for the general case, and

$$\frac{\overline{x_R^2}}{x_S^2} = \frac{W(\hat{r})}{E_S} G_u(\hat{f}_d) \quad (29)$$

when

$$\Phi = \delta(f_d)$$

Since both  $W$  and  $E_S$  vary identically with both the transmitted energy and range, their ratio depends only upon the ratio of the cross section density of the reverberation scatterers to the cross section of the target. The only term in Equations (28) or (29) that depends on the waveform  $u(t)$  is  $G_u$ . This function then determines the ability of a matched filter system to detect and classify targets in a reverberation-limited environment and, therefore, will be called the reverberation function. It varies with the amount of target doppler shift,  $\hat{f}_d$ . If the reverberation is not at zero doppler, the argument of the reverberation function  $G_u$  should be interpreted as the difference between target doppler and reverberation doppler. To indicate this more general case, the notation  $G_u(\nu)$  shall be used in place of  $G_u(f_d)$ . It can be seen from Equation (29) that a low value of  $G_u$  indicates good performance in reverberation.

The reverberation function may be calculated from Equation (25), or from one of the following equivalent expressions

$$G_u(\nu) = \int |U(f)|^2 |U(f + \nu)|^2 df \quad (30)$$

$$= \int |A_u(\tau, 0)|^2 e^{-j 2\pi \nu \tau} d\tau \quad (31)$$

If the waveform  $u(t)$  is not normalized according to Equation (2), then the right-hand sides of Equations (25), (30), and (31) must be divided by  $|A_u(0, 0)|^2$  or, equivalently, by either

$$\int |u(t)|^2 dt^2 \quad \text{or} \quad \int |U(f)|^2 df^2 \quad (32)$$

As a consequence of the Uncertainty Principle, Equation (19), the reverberation function  $G_u(\nu)$  must also satisfy a waveform - invariant relation,

$$\int G_u(\nu) d\nu = 1 \quad (33)$$

for all normalized waveforms,  $u(t)$ . From Equation (30) it is noted that  $G_u(\nu)$  depends only upon the spectral distribution of the transmitted energy,  $|U(f)|^2$ . To get a feeling for the effect of the signal bandwidth on the function  $G_u(\nu)$ , consider a class of signals whose spectral energy distribution is given by,

$$|U(f)|^2 = \frac{1}{B} \left| \frac{\sin \frac{\pi f}{B}}{\frac{\pi f}{B}} \right|^2 \quad (34)$$

This has the same form as the spectral energy distribution of a rectangular pulse of duration  $1/B$ . It is assumed that the waveform duration is  $T$  seconds, and the dependence upon  $T$  shall be indicated by writing Equation (34) as

$$|U_D(f)|^2 = \frac{T}{D} \left| \frac{\sin \frac{\pi f T}{D}}{\frac{\pi f T}{D}} \right|^2 \quad (35)$$

where  $D = BT$  (bandwidth-time product).

When  $D = 1$ , the waveform will be a simple pulse,  $T$  seconds long. For larger values of  $D$ , the waveform is modulated or coded with  $D$  degrees of freedom. The reverberation functions corresponding to Equation (35) are easily shown to be

$$G_D(\nu) = \frac{4T}{D} \cdot \frac{1}{\left(\frac{2\pi \nu T}{D}\right)^2} \left[ 1 - \frac{\sin \frac{2\pi \nu T}{D}}{\frac{2\pi \nu T}{D}} \right] \quad (36)$$

These functions are plotted in Figure 3, against the abscissa,  $\nu T$ , which is the doppler shift of the target with respect to the reverberation, expressed in units of  $1/T$ , the doppler resolution of the waveform. For low values of doppler shift, the reverberation functions decrease directly with the dimensionality,  $D$ , of the waveform. This is to be expected since, if the target is not resolvable from the reverberation in doppler, the signal-to-reverberation ratio can only be improved by increasing the range resolution and, therefore, the bandwidth of the waveform. On the other hand, for large doppler shifts,  $G_D(\nu)$  at first increases with increasing  $D$  and then decreases. The bandwidth-time product,  $D$ , required to equal the performance of the simple pulse for a large value of  $\nu T$  is given approximately by

$$D \approx \frac{2\pi^2}{3} (\nu T)^2 \quad (37)$$

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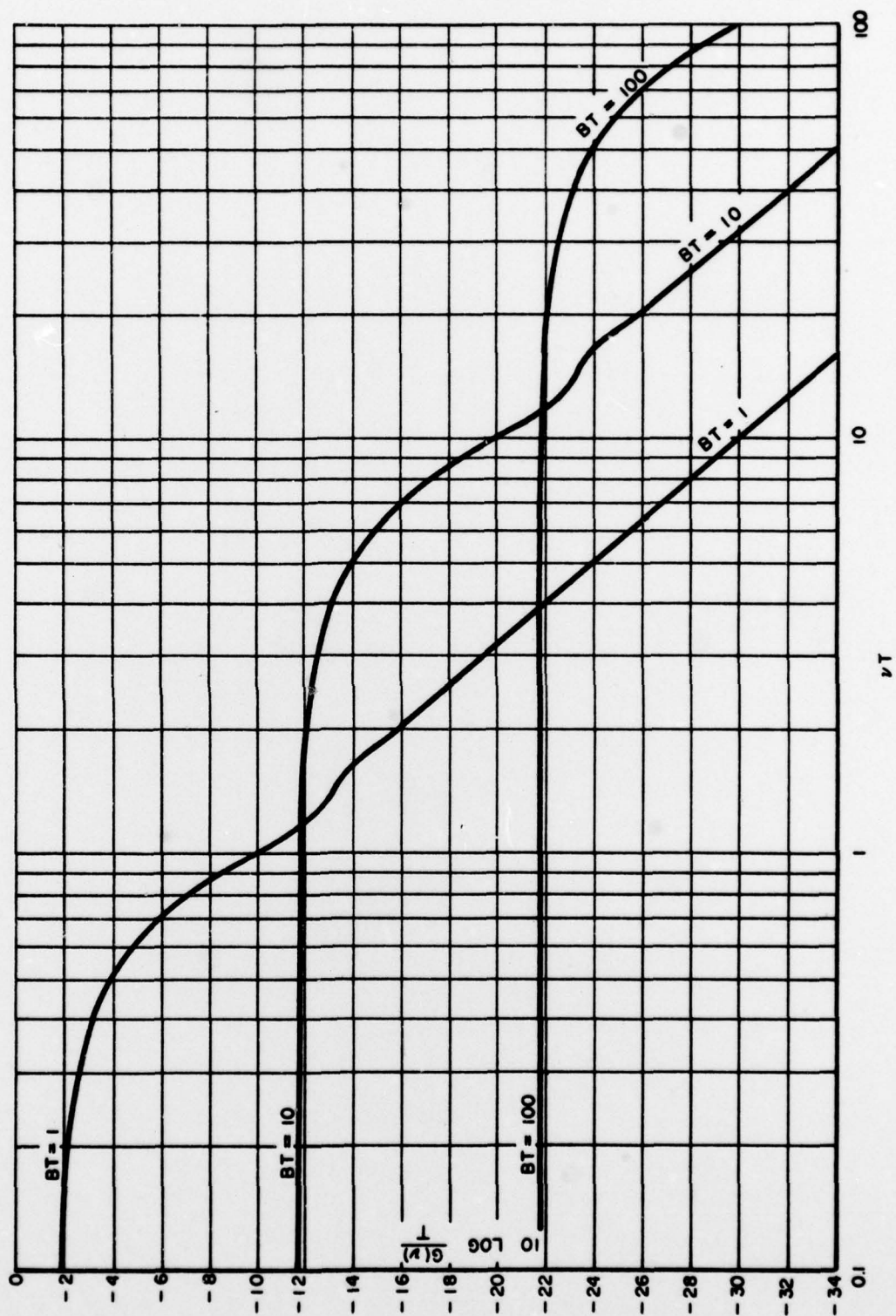


Figure 3. Reverberation Functions of Coded Waveforms with Bandwidth-Time Products of 1, 10, and 100

If this value of BT can be achieved for the largest doppler shift of interest, then it is clear that a large BT waveform is the best choice for combatting reverberation. This is fortunate, since a large BT waveform is also desirable for obtaining detailed range information.

### C. RANDOM WAVEFORMS

In the preceding discussions it has been seen that waveforms having high dimensionality (bandwidth-time) products are very desirable for target classification. Wide bandwidth alone, however, does not insure that a waveform will have a desirable form of ambiguity function. The linear-sweep FM waveform has a wide bandwidth and yet its ambiguity function indicates poor resolution of targets separated in both range and doppler simultaneously. One approach to obtaining a "thumb-tack" ambiguity function is to use a random waveform, that is, a waveform produced by a random or pseudo-random process.

In order to discuss and describe the properties of such waveforms in general, statistical language must be used. The exact ambiguity function of an unknown random waveform cannot be determined, but, if the statistics of the process that determined the waveform are known, the average or expected value of the ambiguity function can be found.

#### 1. Gaussian Noise Waveforms

Consider first the class of waveforms that are formed by gating the output of a random Gaussian noise generator as shown in Figure 4.

The noise at the input to the gate (see Figure 4) has a complex envelope,  $n(t)$ , whose power spectrum, shaped by the band-pass filter, will be  $N_0 |H(f)|^2$ . The autocorrelation function of  $n(t)$  can be found using the Wiener-Khintchin theorem which gives

$$\begin{aligned} R(\tau) &= E \ n(t) \ n^*(t-\tau) \\ &= N_0 \int |H(f)|^2 e^{j 2\pi f \tau} df \end{aligned} \quad (38)$$

It is assumed that the amplitude of  $H(f)$  is adjusted so that  $R(0) = 1$ . This requires that

$$N_0 \int |H(f)|^2 df = 1 \quad (39)$$

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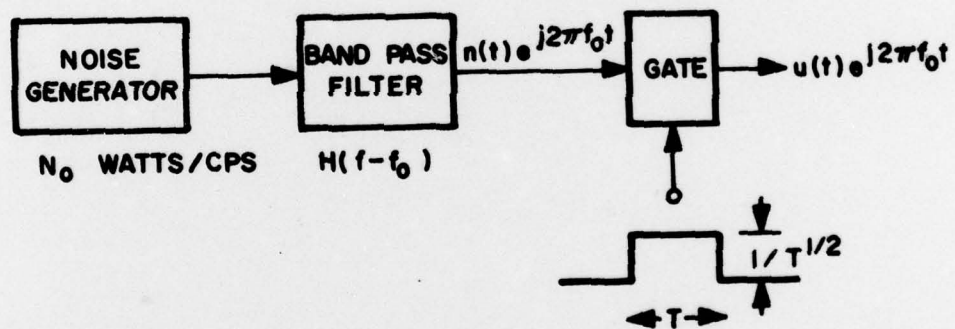


Figure 4. Random Waveform Generator

The waveform at the gate output is the transmitted waveform,  $u(t)$ , which may be described as

$$u(t) = \begin{cases} \frac{n(t)}{T^{1/2}}, & \text{for } |t| \leq T/2 \\ 0, & \text{for } |t| > T/2 \end{cases} \quad (40)$$

The ambiguity function of  $u(t)$  is defined to be

$$A_u(\tau, \nu) = \int_{-T/2}^{T/2-\tau} u(t) u^*(t + \tau) e^{-j 2\pi \nu t} dt \quad (41)$$

and therefore, the expectation or statistical average of  $A(\tau, \nu)$  will be

$$E\{A_u(\tau, \nu)\} = \int_{-T/2}^{T/2-\tau} E\{u(t) u^*(t + \tau)\} e^{-j 2\pi \nu t} dt \quad (42)$$

Using Equations (38) and (40)

$$E\{u(t) u^*(t + \tau)\} = \begin{cases} \frac{1}{T} R^*(\tau), & \text{for } |\tau| \text{ and } |t + \tau| < T/2 \\ 0, & \text{otherwise} \end{cases} \quad (43)$$

which enables evaluation of Equation (42); the result is

$$\begin{aligned} E\{A_u(\tau, \nu)\} &= R^*(\tau) e^{j \pi \nu \tau} \frac{T - |\tau|}{T} \frac{\sin \pi \nu (T - |\tau|)}{\pi \nu (T - |\tau|)} \\ &= R^*(\tau) A_T(\tau, \nu) \end{aligned} \quad (44)$$

where  $A_T(\tau, \nu)$  is the ambiguity function of a simple pulse,  $T$  seconds long.

Assume now that the bandwidth of the filter  $H(f)$  is much broader than  $1/T$ , so that  $u(t)$  has a high bandwidth-time product. In this case, the width of the correlation function  $R(\tau)$ , will be very small compared to  $T$ . As a result, the range dependence and range resolution of the expected ambiguity function,  $E A_u$ , will be determined essentially by the shape and width of  $R(\tau)$ . The doppler dependence of  $E [A_u]$  is determined entirely by the doppler dependence of  $A_T(\tau, \nu)$ , which has a doppler resolution of  $1/T$ .

Although the expected or average ambiguity function calculated above gives some indications of the resolution performance of a randomly selected waveform, it is an incomplete description and in some respects misleading. It is known that the ambiguity function of any particular waveform,  $u(t)$ , must satisfy the Uncertainty Principle. The average ambiguity function given by Equation (44), however, does not satisfy the Uncertainty Principle, since

$$\iint |R(\tau)|^2 |A_T(\tau, \nu)|^2 d\tau d\nu < \iint |A_T(\tau, \nu)|^2 d\tau d\nu = 1 \quad (45)$$

A statistical measure of the resolution performance of a randomly selected waveform,  $u(t)$ , that does satisfy the Uncertainty Principle can be obtained by finding the expected value or statistical average of  $|A_u(\tau, \nu)|^2$ . This makes more sense physically since the signals that are usually of concern are those that add power-wise rather than voltage-wise.

The expectation of  $|A_u|^2$  can be derived as follows

$$E\{|A_u(\tau, \nu)|^2\} = \int_{-T/2}^{T/2-\tau} \int_{-T/2}^{T/2-\tau} E\{u(t) u^*(t + |\tau|) u^*(s) u(s + |\tau|)\} e^{-j2\pi\nu(t-s)} dt ds \quad (46)$$

$$= \frac{1}{T^2} \int_{-T/2}^{T/2-|\tau|} \int_{-T/2}^{T/2-|\tau|} |R(\tau)|^2 + |R(t-s)|^2 e^{-j2\pi\nu(t-s)} dt ds \quad (47)$$

$$= |R(\tau)|^2 |A_T(\tau, \nu)|^2 + \frac{1}{T^2} \int_{\frac{T-|\tau|}{2}}^{\frac{T+|\tau|}{2}} \int_{\frac{T-|\tau|}{2}}^{\frac{T+|\tau|}{2}} |R(t-s)|^2 e^{-j2\pi\nu(t-s)} dt ds \quad (48)$$

$$= |R(\tau)|^2 |A_T(\tau, \nu)|^2 + \frac{1}{T^2} \int_{-(T-|\tau|)}^{T-|\tau|} |R(x)|^2 (T-|\tau|-|x|) e^{-j2\pi\nu x} dx \quad (49)$$

For waveforms having large BT products,  $|R(x)|^2$  will have a small spread in "x" compared to  $T-|\tau|$  (for most values of  $\tau$ ), and hence the following approximation to Equation (49) may be obtained for use of Equations (38) and (39).

$$E\{|A_u(\tau, \nu)|^2\} \approx |R(\tau)|^2 |A_T(\tau, \nu)|^2 + \frac{T-|\tau|}{T} \Gamma(\nu) \quad (50)$$

where

$$\Gamma(\nu) = \frac{\int |H(f)|^2 |H(f+\nu)|^2 df}{\int |H(f)|^2 df} \quad (51)$$

Assuming further that  $|H(f)|^2$  is approximately rectangular in shape with a bandwidth of  $B$  cps, Equation (50) can be simplified to

$$E \left\{ |A(\tau, \nu)|^2 \right\} \approx |R(\tau)|^2 |A_T(\tau, \nu)|^2 + \frac{1}{BT} \left( 1 - \frac{|\tau|}{T} \right) \left( 1 - \frac{|\nu|}{B} \right) \quad (52)$$

The first term in Equation (52) is the square of the value obtained previously for the average value of  $A_u$ . It represents the central spike in the ambiguity function. Its range resolution corresponds to the bandwidth of  $u(t)$  and its doppler resolution to the time duration of  $u(t)$ . The second term is the undesirable part of the ambiguity function, that is, the side lobes or self-noise of the waveform. In the  $\tau, \nu$  plane it extends over  $\pm T$  in  $\tau$  and  $\pm B$  in  $\nu$  with a peak height of  $1/BT$  and a total volume of unity.

The performance of the randomly selected waveform against reverberation is considered next. It has been seen that for an explicitly defined waveform this is determined by the reverberation function

$$G_u(\nu) = \int_{-T}^T |A_u(\tau, \nu)|^2 d\tau \quad (53)$$

For a randomly selected waveform, the average performance in reverberation is determined by the expected value of  $G_u$

$$E \left\{ G_u(\nu) \right\} = \int_{-T}^T E \left\{ |A_u(\tau, \nu)|^2 \right\} d\tau \quad (54)$$

Using Equation (49), this becomes

$$\begin{aligned} E \left\{ G_u(\nu) \right\} &= \int_{-T}^T |R(\tau)|^2 |A_T(\tau, \nu)|^2 d\tau \\ &\quad + \frac{1}{T^2} \int_{-T}^T \int_{-(T-|\tau|)}^{T-|\tau|} |R(x)|^2 (T-|\tau|-|x|) e^{-j2\pi\nu x} dx d\tau \end{aligned} \quad (55)$$

$$E \{ G_u(\nu) \} = \int_{-T}^T |R(\tau)|^2 \frac{\sin^2 \pi \nu (T-|\tau|)}{(\pi \nu T)^2} d\tau + \frac{1}{T^2} \int_{-T}^T |R(\tau)|^2 (T-|\tau|)^2 e^{-j2\pi \nu \tau} d\tau \quad (56)$$

Assuming, as before, that  $u(t)$  is a large BT product waveform so that  $|R(\tau)|^2$  is a delta-like function compared to the other terms in Equation (56), the following approximation is obtained

$$E \{ G_u(\nu) \} = \frac{\sin \pi \nu T}{\pi \nu T}^2 \int |R(\tau)|^2 d\tau + \int |R(\tau)|^2 e^{-j2\pi \nu \tau} d\tau \quad (57)$$

Both integrals in Equation (57) can be expressed in terms of the function  $\Gamma(\nu)$  defined by Equation (51), so that

$$E \{ G_u(\nu) \} \approx \frac{\sin \pi \nu T}{\pi \nu T}^2 \Gamma(0) + \Gamma(\nu) \quad (58)$$

If the power spectrum of the noise process is approximately rectangular in shape with a bandwidth of  $B$  cps, the expected value of  $G_u$  becomes

$$E \{ G_u(\nu) \} \approx \frac{T}{BT} \left\{ 1 - \frac{\nu T}{BT} + \left[ \frac{\sin \pi \nu T}{\pi \nu T} \right]^2 \right\} \quad (59)$$

It is interesting to compare this result with the reverberation function of a "definite" waveform that is  $T$  seconds long and has a rectangular spectral distribution of energy of the form

$$|U(f)|^2 = \begin{cases} 1/B & \text{for } |f| \leq B/2 \\ 0 & \text{for } |f| > B/2 \end{cases} \quad (60)$$

Substituting Equation (60) in Equation (27)

$$G'_u(\nu) = \frac{T}{BT} \left[ 1 - \frac{\nu T}{BT} \right] \quad (61)$$

Comparing this with Equation (59), it is seen that the random waveform has approximately 3 db poorer performance against reverberation for values of  $\nu T < \frac{1}{2}$ ; for larger values of  $\nu T$  the random waveform does as well as the "definite" waveform.

## 2. Random Binary-Phase Modulation

The random waveforms considered in the foregoing discussion were produced by gating  $T$  seconds of band-pass Gaussian noise. As a result, these waveforms were modulated in both amplitude and phase within the  $T$ -second gate period. It is very desirable, however, to use waveforms that have a constant amplitude during the gate period and are coded or modulated in phase only. Transmitter efficiency is usually higher with constant amplitude waveforms and, more important, maximum energy is obtained for a given pulse length since cavitation limits the peak power delivered to the water.

A simple example of a randomly, phase-modulated waveform is a carrier whose phase is changed by either  $0^\circ$  or  $180^\circ$  at periodic intervals. Such a waveform could be generated by multiplying a sinusoid carrier by a random square wave, such as shown in Figure 5. If the waveform of Figure 5 is called  $u(t)$ , the complex representation of the complete signal would be  $u(t) \exp j2\pi f_c t$ . To get a mathematically tractable formulation of  $u(t)$ , note that  $u(t)$  may be considered to be the sum of a sequence of identical pulses, each of duration  $\lambda$ , with randomly chosen algebraic signs, that is

$$u(t) = \sum_{k=0}^{N-1} x_k p(t - k\lambda) \quad (62)$$

where

$$p(t) = \begin{cases} \frac{1}{\lambda^{1/2}} & \text{for } |t| < \frac{\lambda}{2} \\ 0 & \text{for } |t| > \frac{\lambda}{2} \end{cases} \quad (63)$$

and

$$x_k = \begin{cases} +\frac{1}{N^{1/2}} & \text{with probability } 1/2 \\ -\frac{1}{N^{1/2}} & \text{with probability } 1/2 \end{cases} \quad (64)$$

It shall be assumed that the random variables,  $x_k$ , are independent, so that

$$E \{x_k x_l\} = \begin{cases} 0 & \text{for } k \neq l \\ \frac{1}{N} & \text{for } k = l \end{cases} \quad (65)$$

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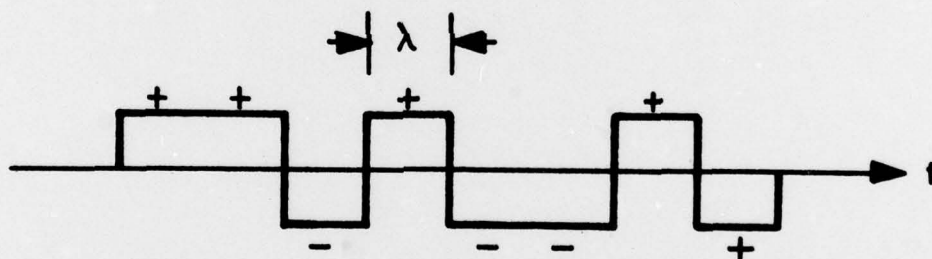


Figure 5. Random Binary Waveform

As before the expected values of the ambiguity function,  $A_u(\tau, \nu)$  shall be calculated, and the expected value of the reverberation function,  $G_u(\nu)$ .

By definition

$$A_u(\tau, \nu) = \int u(t) u^*(t + \tau) e^{-j 2\pi \nu t} dt \quad (66)$$

Substituting from Equation (62)

$$A_u(\tau, \nu) = \sum_k \sum_{\ell=0}^{N-1} x_k x_\ell \int p(t - k\lambda) p(t + \tau - \ell\lambda) e^{-j 2\pi \nu t} dt \quad (67)$$

This can be written very conveniently in terms of the ambiguity function of the basic pulse  $p(t)$ . Thus, letting

$$A_p(\tau, \nu) = \int p(t) p(t + \tau) e^{-j 2\pi \nu t} dt = e^{j \pi \nu \lambda} \frac{\sin \pi \nu (\lambda - |\tau|)}{\pi \nu (\lambda - |\tau|)} \quad (68)$$

Equation (67) can be written as

$$A_u(\tau, \nu) = \sum_k \sum_{\ell=0}^{N-1} x_k x_\ell e^{-j 2\pi k \nu \lambda} A_p[(k-\ell) \lambda + \tau, \nu] \quad (69)$$

The expected value of  $A_u(\tau, \nu)$  is easily found using Equation (65)

$$\begin{aligned} E[A_u(\tau, \nu)] &= \sum_k \sum_{\ell=0}^{N-1} E\{x_k x_\ell\} e^{-j 2\pi k \nu \lambda} A_p[(k-\ell) \lambda + \tau, \nu] \quad (70) \\ &= \frac{1}{N} \sum_{k=0}^{N-1} e^{-j 2\pi k \nu \lambda} A_p(\tau, \nu) \\ &= e^{-j(N-1)\pi \nu \lambda} \frac{\sin N\pi \nu \lambda}{N \sin \pi \nu \lambda} A_p(\tau, \nu) \end{aligned}$$

The expected value of  $|A_u|^2$  is somewhat more difficult to obtain. Starting with

$$\begin{aligned} E\{|A_u(\tau, \nu)|^2\} &= \sum_k \sum_{\ell, r, s=0}^{N-1} E\{x_k x_\ell x_r x_s\} e^{-j 2\pi \nu \lambda (k-r)} \quad (71) \\ &\quad \times A_p[(k-\ell) \lambda + \tau, \nu] A_p^*[(r-s) \lambda + \tau, \nu] \end{aligned}$$

and noting that  $E \{x_k x_r x_s\}$  is either zero or  $1/N^2$ . It will equal  $1/N^2$  if, and only if (a)  $k=l$  and  $r=s$ , or (b)  $k=r$  and  $l=s$ , or (c)  $k=s$  and  $l=r$ . These three conditions are not mutually exclusive since they contain the common condition (d)  $k=l=r=s$ .

$E \{ |A_u|^2 \}$  can be obtained by adding the contributions to the quadruple sum on the right-hand side of Equation (71) under conditions (a), (b), and (c), and then subtracting twice the contribution to the quadruple sum under condition (d). The contributions for each of the four cases are

$$(a) \frac{1}{N^2} \sum_k \sum_{r=0}^{N-1} e^{-j2\pi\nu\lambda(k-r)} |A_p(\tau, \nu)|^2 = |A_p(\tau, \nu)|^2 \left[ \frac{\sin N\pi\nu\lambda}{N \sin \pi\nu\lambda} \right]^2 \quad (72)$$

$$(b) \frac{1}{N^2} \sum_k \sum_{l=0}^{N-1} |A_p[(k-l)\lambda + \tau, \nu]|^2 = \sum_{k=-(N-1)}^{N-1} \frac{N-|k|}{N^2} |A_p(k\lambda + \tau, \nu)|^2 \quad (73)$$

$$(c) \frac{1}{N^2} \sum_k \sum_{l=0}^{N-1} e^{-j2\pi\nu\lambda(k-l)} \{A_p[(k-l)\lambda + \tau, \nu]\} \{A_p^*[(l-k)\lambda + \tau, \nu]\} \quad (74)$$

and

$$(d) \frac{1}{N^2} \sum_{k=0}^{N-1} |A_p(\tau, \nu)|^2 = \frac{1}{N} |A_p(\tau, \nu)|^2 \quad (75)$$

The summation for case (c) reduces to  $1/N |A_p(\tau, \nu)|^2$  since

$$A_p(\tau, \nu) = 0 \text{ for } |\tau| > \lambda \quad (76)$$

Assembling the above results

$$E \{ |A_u(\tau, \nu)|^2 \} = \left[ \frac{\sin N\pi\nu\lambda}{N \sin \pi\nu\lambda} \right]^2 |A_p(\tau, \nu)|^2 - \frac{1}{N} |A_p(\tau, \nu)|^2 + \sum_{k=-(N-1)}^{N-1} \frac{N-|k|}{N^2} |A_p(k\lambda + \tau, \nu)|^2 \quad (77)$$

The first term on the right-hand side represents the "central spike" of the ambiguity function. Its width in doppler, determined by the first factor, is  $1/T$  where  $T = N\lambda$ , is the total duration of the waveform. The width in the range

dimension is determined by the second factor  $|A_p|^2$  and, therefore, is approximately  $\lambda$  or  $T/N$ . Note that  $N$  is a measure of the dimensionality or bandwidth-time product of the waveform. The remaining two terms in Equation (77) represent the ambiguity side lobes or self-noise of the waveform. The side lobes have a maximum value of approximately  $1/N$  at  $\tau, \nu = \pm \lambda, 0$  and decrease linearly with increasing  $|\tau|$ .

The expected reverberation function,  $E\{G_u\}$ , is now evaluated. Applying the expectation operator to Equation (25)

$$E\{G_u(\nu)\} = \int E\{|A_u(\tau, \nu)|^2\} d\tau \quad (78)$$

and then using the expression for the integrand given in Equation (77), it is found that

$$E\{G_u(\nu)\} = G_p(\nu) \left\{ \frac{N-1}{N} + \left| \frac{\sin N\pi\nu\lambda}{N \sin \pi\nu\lambda} \right|^2 \right\} \quad (79)$$

where

$$G_p(\nu) = \int |A_p(\tau, \nu)|^2 d\tau = \frac{4\lambda}{(2\pi\nu\lambda)^2} \left\{ 1 - \frac{\sin 2\pi\nu\lambda}{2\pi\nu\lambda} \right\} \quad (80)$$

Substituting this expression in Equation (79), and setting  $T = N\lambda$

$$E\{G_u(\nu)\} = \frac{4T}{N} \frac{1}{\left[ \frac{2\pi\nu T}{N} \right]} \left[ 1 - \frac{\sin \frac{2\pi\nu T}{N}}{\left[ \frac{2\pi\nu T}{N} \right]} \right] \left[ \frac{N-1}{N} + \left| \frac{\sin N\pi\nu\lambda}{N \sin \pi\nu\lambda} \right|^2 \right] \quad (81)$$

Except for the last factor, this result is identical to the result obtained in Equation (36) for the reverberation function of a "definite" waveform having the energy spectral distribution of a rectangular pulse and a dimensionality of  $N$ . The second factor, however, indicates that the average reverberation function for the random waveform is approximately 3 db larger for values of  $\nu T < 1/2$ . This is similar to the result obtained for Gaussian random waveforms.